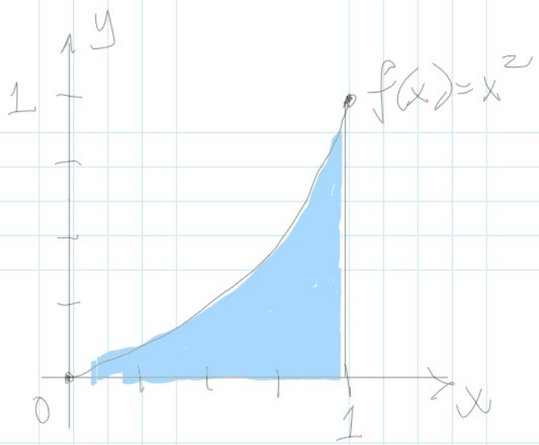
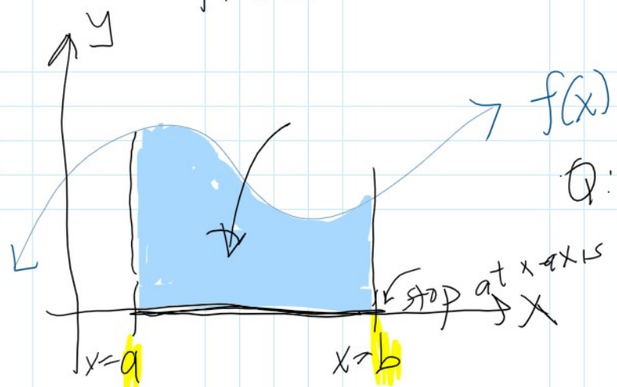
23-Jan 2023

Monday, January 23, 2023 2:43 PM



Area

"Problem Child"



Q: what is the exact area under $f(x)$ between $x=a$ and $x=b$?

can't find exact area ... yet

y

$f(x)=4$

to find exact area of shaded region, use

a 0

lower bound

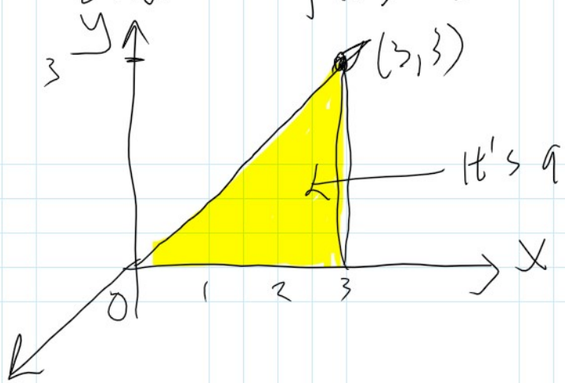
upper bound

to find area of shaded region, use your knowledge of the areas of standard shapes

$A_{\square} = bh$

ex. find exact area under $f(x) = x$ between $x = 0$ and $x = 3$.

sketch $f(x) = x$



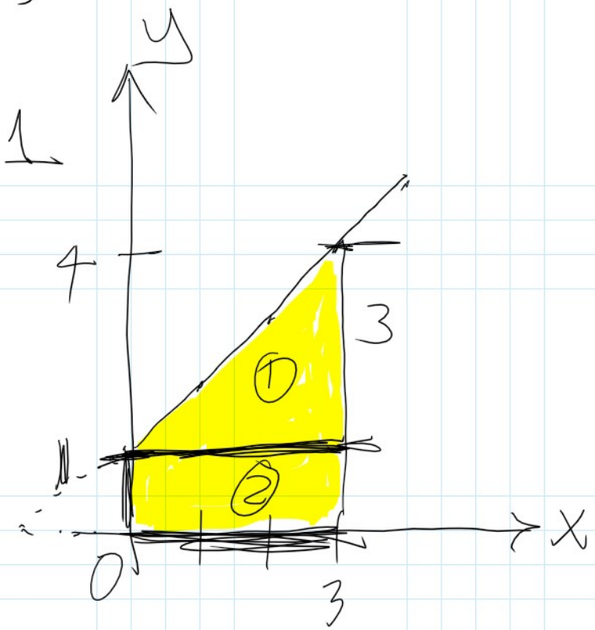
It's a triangle!

$A_{\triangle} = \frac{1}{2}bh$

$= \frac{1}{2}(3)(3)$

$= \frac{9}{2}$

DO: find exact area under $f(x) = x + 1$ between $x = 0$ and $x = 3$

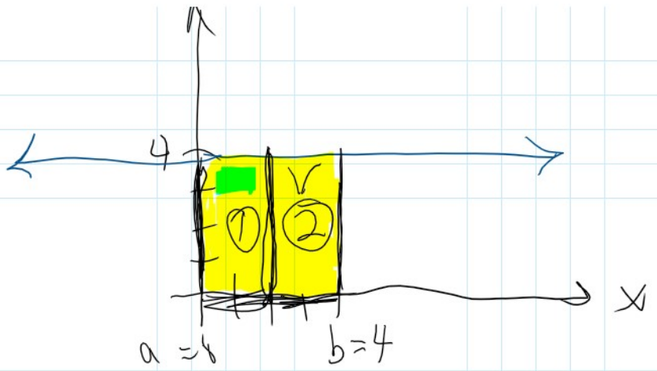


$A_{TOTAL} = A_{①} + A_{②}$

$= \frac{1}{2}(3)(3) + (3)(1)$

$= \frac{9}{2} + 3 \cdot \frac{2}{2}$

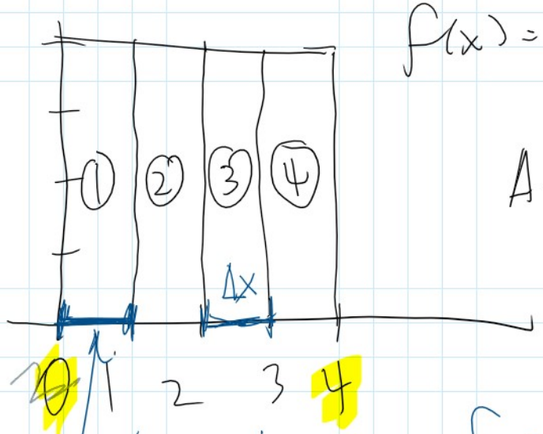
$= \frac{9}{2} + \frac{6}{2} = \boxed{\frac{15}{2}}$



$A_{TOTAL} = A_① + A_② = 16$

$= 2(4) + 2(4)$

↑ base



$f(x) = 4$

$A_{TOT} = A_① + A_② + A_③ + A_④ = 4$

base is same for each rectangle/slice

Δx n is the # of slices

use $f(x) = x^2$ $a = 0$ $b = 1$

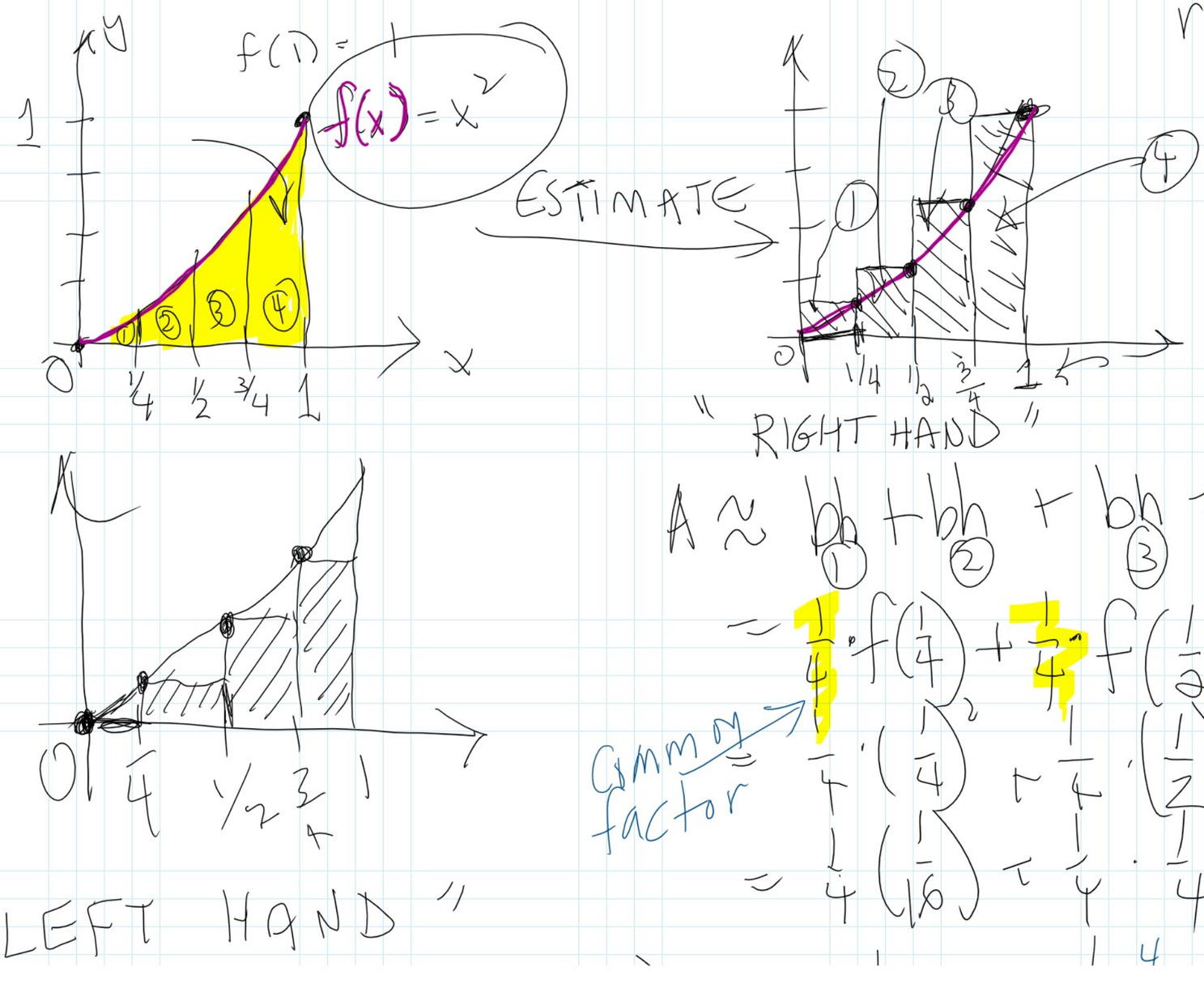
$f(0) = 0$

EXACT AREA $= A_① + A_② + A_③ + A_④$

For now, use rectangle to ESTIMATE exact area under curve

recall $A_{\square} = bh$

$\Delta x \cdot f(\quad)$



$\Delta x = \frac{1}{4}$

$A \approx \underset{①}{bh} + \underset{②}{bh} + \underset{③}{bh} + \underset{④}{bh}$

$= \frac{1}{4} \cdot f\left(\frac{1}{4}\right) + \frac{1}{4} f\left(\frac{1}{2}\right) + \frac{1}{4} f\left(\frac{3}{4}\right) + \frac{1}{4} \cdot f(1)$

Gimmy factor

$= \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{2}\right)^2 + \frac{1}{4}\left(\frac{3}{4}\right)^2 + \frac{1}{4}(1)$

$= \frac{1}{4}\left(\frac{1}{16}\right) + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{9}{16} + \frac{1}{4}$

$\frac{1}{64} \quad \frac{4}{64} \quad \frac{9}{64} \quad \frac{16}{64}$

"LEFT HAND"

$$A \approx b(h_1 + h_2 + h_3 + h_4) = \frac{1}{4}\left(f(0) + f\left(\frac{1}{4}\right) + f\left(\frac{7}{2}\right) + f\left(\frac{3}{4}\right)\right)$$

$$= \frac{1}{4}\left(\quad\right)$$

$$\approx \frac{7}{32} \approx 0.22$$

$$\frac{1}{64} + \frac{1}{16}\cdot\frac{4}{4} + \frac{9}{64} + \frac{1}{4}\cdot\frac{16}{16}$$

$$\frac{1+4+9+16}{64} = \frac{30}{64} = \frac{15}{32} \approx 0.47$$

INSIDER INFO:

EXACT AREA $f(x) = x^2$ btwn $x=0$ $x=1$ is $\frac{1}{3} \approx .33$

$$\frac{7}{32} < \frac{47}{\quad}$$

next time:

$.22 < \frac{1}{3} < .47$

L_4 R_4

next time:
explore how to find
better estimations